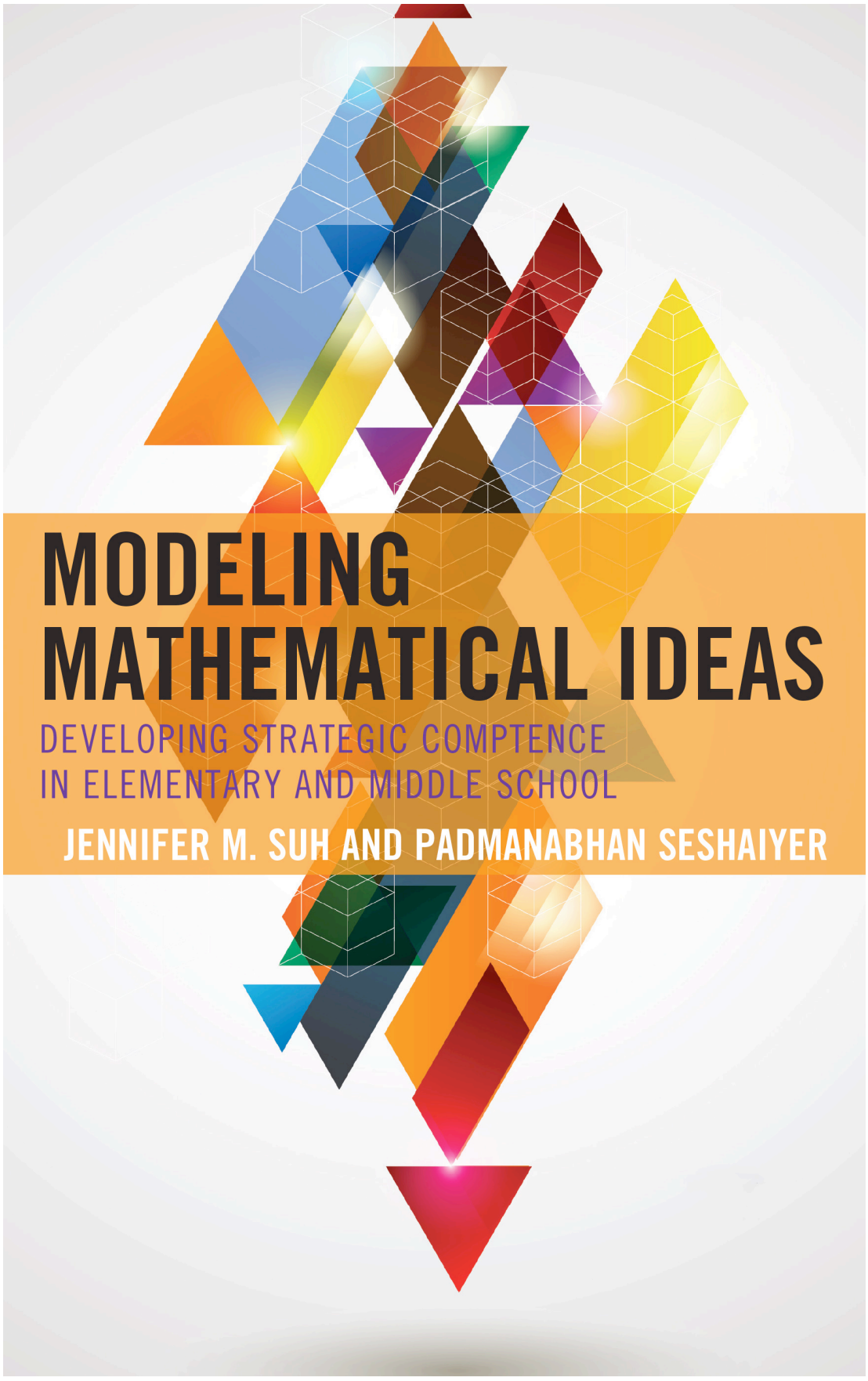
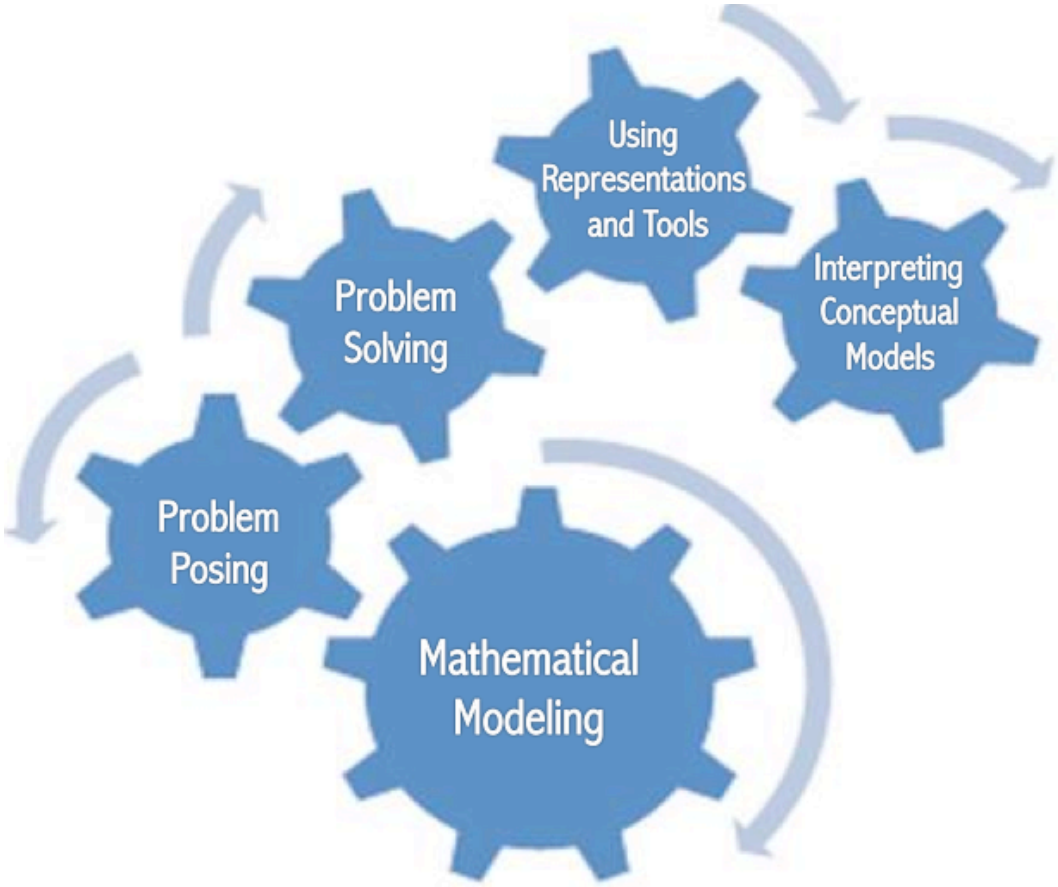

MODELING
MATHEMATICAL IDEAS
DEVELOPING STRATEGIC COMPTENCE
IN ELEMENTARY AND MIDDLE SCHOOL
JENNIFER M. SUH AND PADMANABHAN SESHAIYER



Developing Strategic Competence through Modeling Mathematical Ideas include the application of mathematics for 1) problem solving; 2) problem posing; 3) mathematical modeling; 4) the flexible use of representational models, tools, technology and manipulatives to solve problems and communicate mathematical understanding; and 5) the deep understanding of conceptual models critical to understanding a specific mathematics topic.

MMI Toolkit 1.2 Performance-based Assessment

Performance based assessment: The Classic Handshake Problem	
NCTM: Algebra (Mathematical Problem Solving) Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities	
1. If everyone at your table shakes hands with everyone else, how many handshakes would there be? 2. If everyone in your class shakes hands with everyone else, how many handshakes would there be? 3. What if there were 100 people in the room? 4. At a birthday party, each guest shakes hands with every guest. If 190 different handshakes take place, how many guests were at the party?	
Anticipated Students Response and performance	Tools & Technology
Developing An Algebraic Habit of Mind (Driscoll, 1999) 1. Abstracting from computation 2. Doing and undoing 3. Building a rule from patterns	Manipulatives http://completecenter.gmu.edu/java/handshake/index.html
Signposts for evaluation: Did the student use: ☐ pictures, charts, graphs, or t-tables with supporting explanation ☐ a written explanation with detailed sentences ☐ the equation or number sentence ☐ the answer (Is the answer reasonable? Why or why not?) ☐ the solution to relate to other situations	Teacher Notes: **(1) Understanding-** **(2) Computing-** **(3) Applying-** **(4) Reasoning-** **(5) Engaging-**

MMI Toolkit 2. UCARE Rubric to assess mathematics proficiency

Student name	Comments (Supporting Evidence)
Understanding (Conceptual Understanding)	
o Understands the problem and task o Makes connection to similar problems o Uses and connects models and multiple representations o Reasons abstractly and quantitatively*	
Computing (Procedural Fluency)	
o Looks for and makes use of structure* o Performs accurate computation & attends to precision* o Uses proper algorithm o Demonstrates flexibility in computation	
Applying (Strategic Competence)	
o Formulates and carries out a plan o Poses similar problems o Models with mathematics* o Makes assumptions and constraints to simplify a real world problem o Solves problem using appropriate math and strategies o Uses appropriate tools strategically*	
Reasoning (Adaptive Reasoning)	
o Justifies responses logically o Reflects on and explains procedures o Explains concepts clearly o Constructs viable arguments and critiques the reasoning of others*	
Engaging (Productive Disposition)	
o Tackles difficult tasks o Perseveres o Shows confidence in one's ability o Collaborates/Shares ideas	

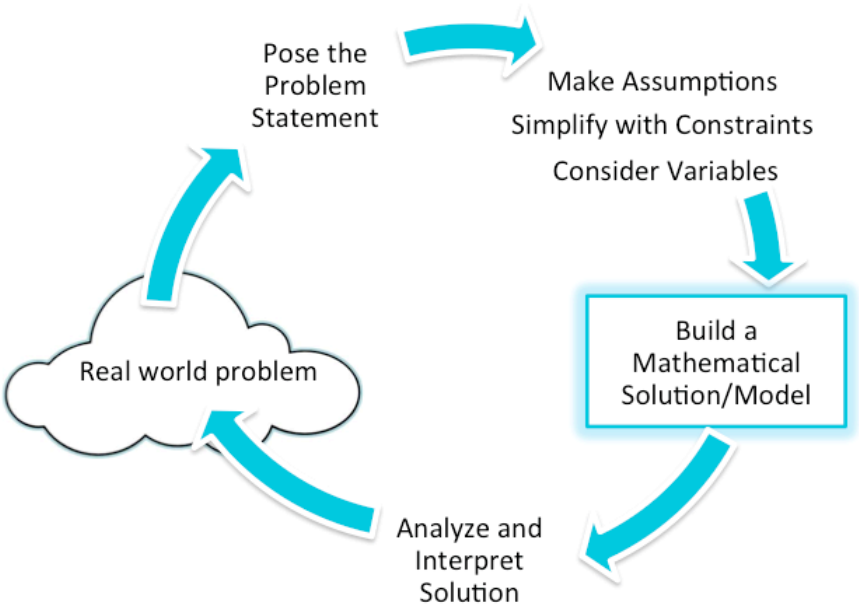
Overall Assessment:

MMI Toolkit 3.1 The Math Modeling Cycle

Mathematical Modeling in the Elementary Grades

Mathematical Modeling involves posing mathematical problems in authentic real life contexts that are relatable to students' personal interests, knowledge and skills.

Mathematical Modeling enables students to use mathematics to help make decisions (i.e., optimize, predict and determine the meaningful solutions to the problem).



Math Modeling Process- Modified from the SIAM- Moody's Mega Math Challenge website)

1. **Pose the Problem Statement: Is it real-world and does it require math modeling? What mathematical questions come to mind?**

2. **Make Assumptions, Define, and Simplify: What assumptions do you make? What are the constraints that help you define and simplify the problem?**

3. **Consider the Variables: What variables will you consider? What data/information is necessary to answer your question?**

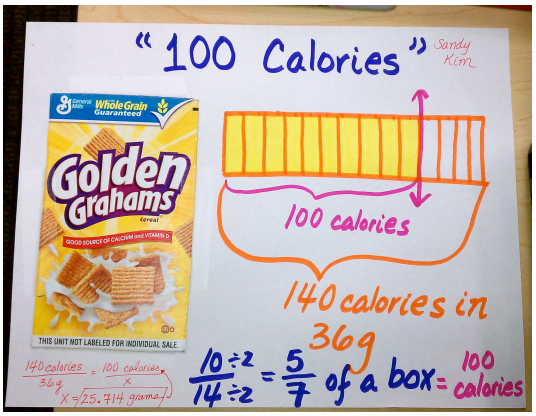
4. **Build Solutions: Generate solutions.**

5. **Analyze and Validate Conclusions: Does your solutions make sense? Now, take your solution and apply it to the real world scenario. How does it fit? What do you want to revise?**

6. **Present and Justify the Reasoning for Your Solution**

MMI Toolkit 4. Sharing Math Happenings

Share a math happening!



OBJECTIVE: Share a real-life event (math happening)
and pose a question that can be answered using the information given in the story. Illustrate the number sentence by drawing a picture.

MATERIALS: real world math materials, artifacts, and or manipulatives to engage students

BACKGROUND INFORMATION: Math happenings occur daily in all of our lives. The math happening lessons serve as a framework for teaching many mathematical concepts within the context of real-life math events. The teacher's role in the math happening lesson is:

- to encourage students to share stories about events that actually happen to them
- to interpret, translate, and represent these stories mathematically, using appropriate materials
- to introduce other math concepts for which students are ready.

PROCEDURE:
The teacher begins by grouping students on the floor and asking them to tell a real-life situation that involved math. This can be done by using Plan A, Plan B, or Plan C.

- Plan A - What math happened to you? Tell us about it. (Generate questions about given data.)
- Plan B - Tell me what you did last night, yesterday, or this weekend. (Listen to the event. Probe to gain enough information to make a math story and ask a question.)
- Plan C - Math happened to me. Let me tell you about it. (Tell the story. Say what you're trying to find out. Ask the question.)

After a story has been shared, the teacher and students can model the story using real objects.

For example, if the story is about planning a table arrangement for a family feast to see how many people can sit around different arrangements, desks could be be used to act out the story. If real objects are unavailable, representative objects can be used (color tiles instead of tables). The teacher then writes a math sentence on a sentence strip to go with the story. (As the "math happening" becomes more of a classroom routine, the students can begin to write the math sentence.) The answer to the math sentence should also be written. The student who shares the math story can later illustrate the story that corresponds to the number sentence. After the teacher has modeled many stories (over time) using materials, all students can represent problems with semi-concrete materials, drawings, numbers and words.

CCSS M STANDARD 4: MODEL WITH MATH

- **Model with mathematics. From Standard 4 of CCSS-M**

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose. (NGACBP & CCSSO, 2010)

- Share a mathematical modeling task that affords these mathematical thinking activities.
- How do you promote modeling with math in your classroom?

A Math Happening B: Planning for the Table Tennis Championship

Next week, there will be a table tennis championship. Plan how to organize the league, so that the tournament will take the shortest possible time. Put all the information on a poster so that the players can easily understand what to do. -Problem from the PRIMAS website

The following six steps were modified from the steps as defined by the Society for Industrial and Applied Mathematics SIAM- Moody's Mega Math Challenge, which is a national mathematical modeling contest for high school students sponsored by The Moody's Foundation (see http://m3challenge.siam.org/resources/modeling-handbook)

1. **Posing the Problem Statement:** Is it real-world and does it require math modeling? What mathematical questions come to mind?

2. **Making Assumptions to Define, and Simplify the Real World Problem:** What assumptions do you make? What are the constraints that help you define and simplify the problem?

3. **Considering the Variables:** What variables will you consider? What data/information is necessary to answer your question?

4. **Building Solutions:** Generate solutions.

5. **Analyzing and Validating their Conclusions:** Does your solutions make sense? Now, take your solution and apply it to the real world scenario. How does it fit? What do you want to revise?

6. **Presenting and Justifying the Reasoning for Your Solution**

EFFECTIVE MATHEMATICS TEACHING PRACTICES

1. Establish mathematics **goals** to focus learning.
2. Implement **tasks** that promote reasoning and problem solving.
3. Use and connect mathematical **representations**.
4. Facilitate meaningful mathematical **discourse**.
5. Pose purposeful **questions**.
6. Build procedural **fluency** from conceptual **understanding**.
7. Support productive **struggle** in learning mathematics.
8. Elicit and use **evidence** of student thinking.

Math Goals	What might be the math learning goals?
Tasks & Representations	What representations might students use in reasoning through and solving the problem?
Discourse & Questions	How might we question students and structure class discourse to advance student learning?
Fluency from Understanding	How might we develop student understanding to build toward aspects of procedural fluency?
Struggle & Evidence	How might we check in on student thinking and struggles and use it to inform instruction?